Groundwaterflow and Transport

Characteristic Values of Water and Soil

Water *homogenous, small variability for groundwater*

Density 1000 kg/m^3

Viscosity 1,31 10^{-6} m^2/s

Compressibility 4,789 10^{-10} m^2/N

Soil *very heterogenious, high variability*

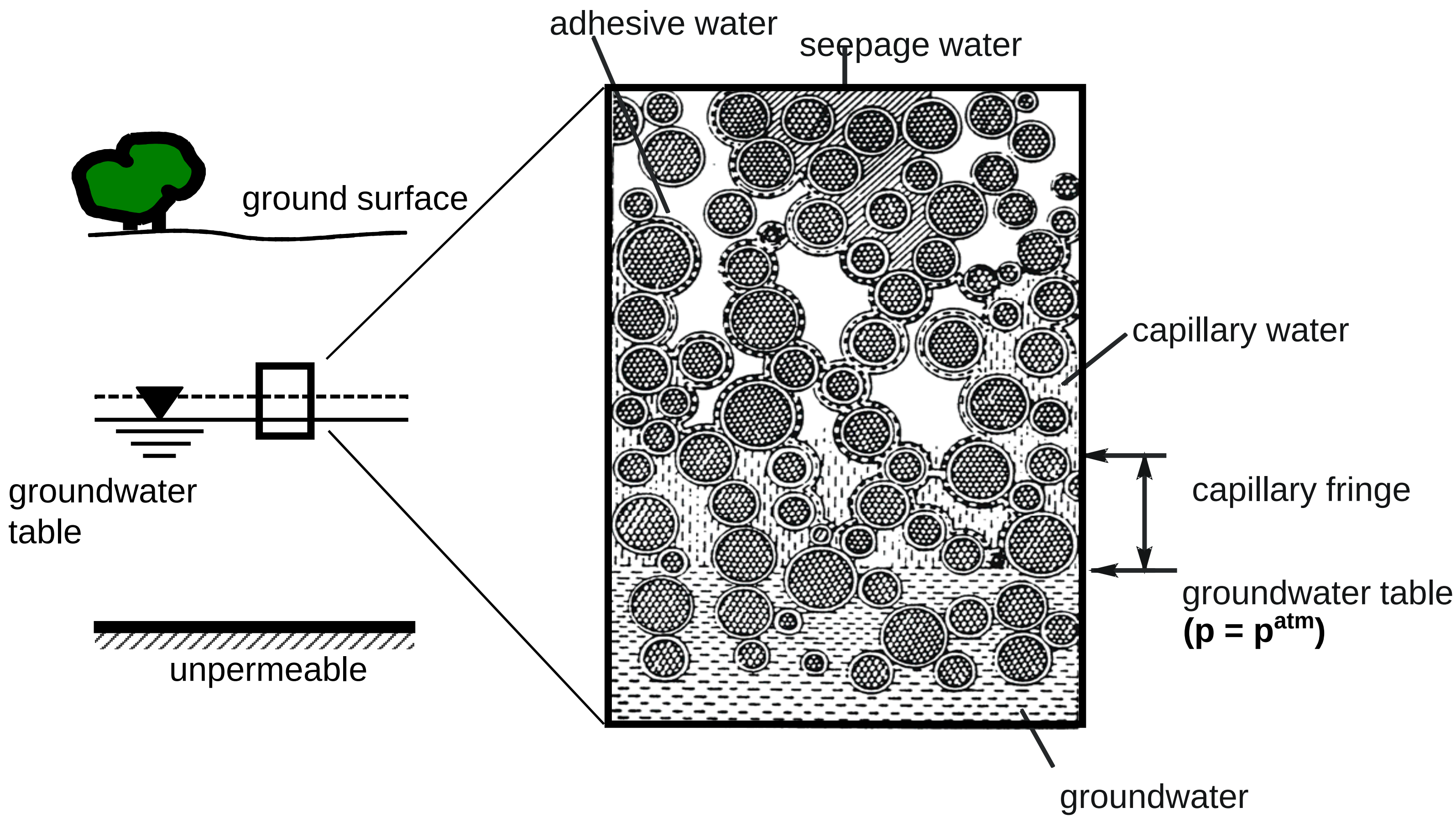
Permeability (depends on form, type, number and saturation of pores and physical characteristics of grains)
e.g middle sand 10^{-3}-10^{-4} m/s, clay < 10^{-9} m/s

Saturation of water 0-100% of porevolume

Porosities (total, flow and storage effectiv),
e.g. utilisable porosity clay < 5%, middle sand 12-25 %

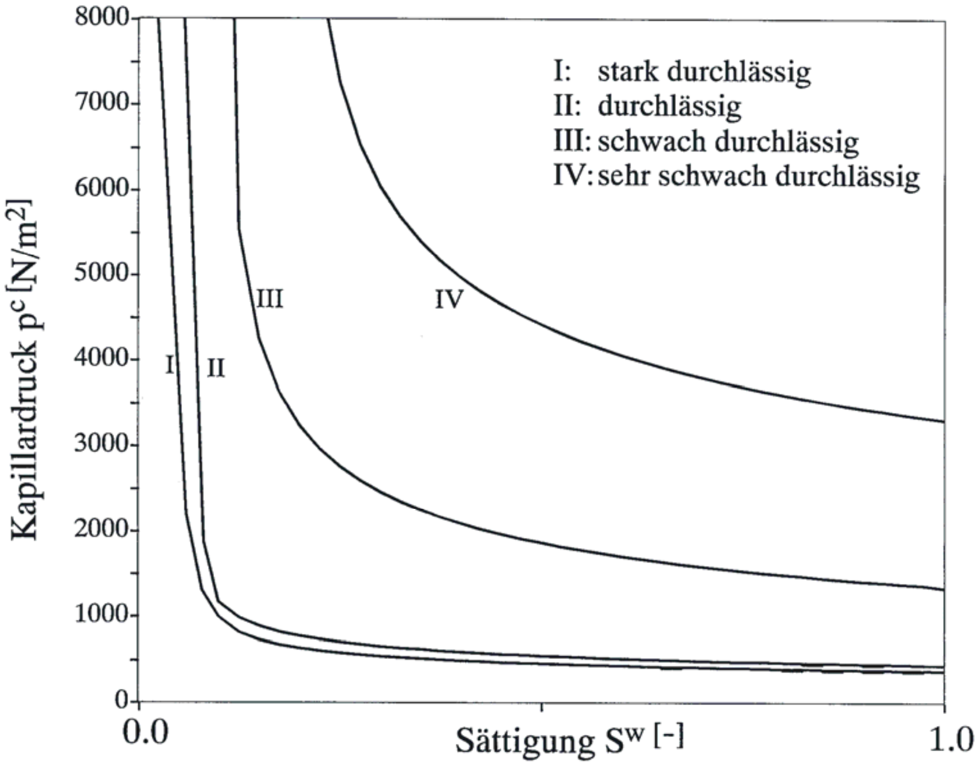
compressibility of granular structure,
e.g. sandstone approx. 2 10^{-8} m^2/N

Saturation

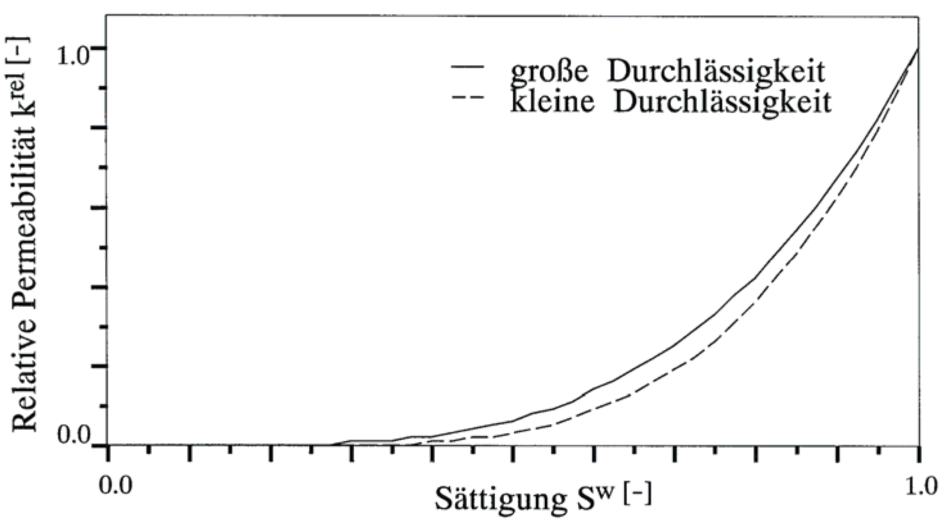


$$S^w = \frac{\text{water volume}}{\text{pore volume}} \quad [\%]$$

Saturation is a macroscopic parameter. In microscale a pore is filled by water or air.

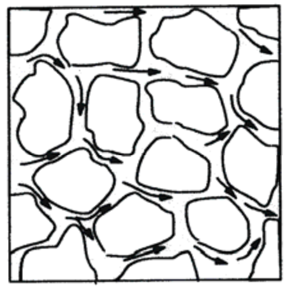


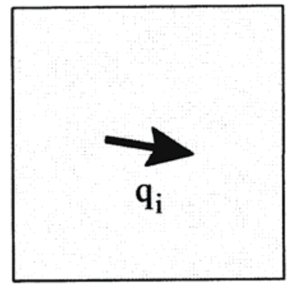
capillary pressure-saturation



relative permeability

Darcy‘s Law – Flow in porous aquifer





Homogenization of porous media leads to the definition of a (fictitious) water velocity, called Darcy velocity.
For one dimensional flow (e.g. in a sand column) is the Darcy velocity equal to water volume per time unit (Q), devided by cross section of the filter (A).

$$q = \frac{Q}{A} = -k_f \cdot \frac{\partial h}{\partial x}$$

$$\left[\frac{m}{s}\right] = \left[\frac{m^3/s}{m^2}\right] = \left[\frac{m}{s}\right] \cdot \left[\frac{m}{m}\right]$$

$\frac{\partial h}{\partial x}$ = gradient of potential head

k_f = permeability

The Darcy velocity is <u>not</u> equal to the particle velocity.

Generalized Darcy's Law

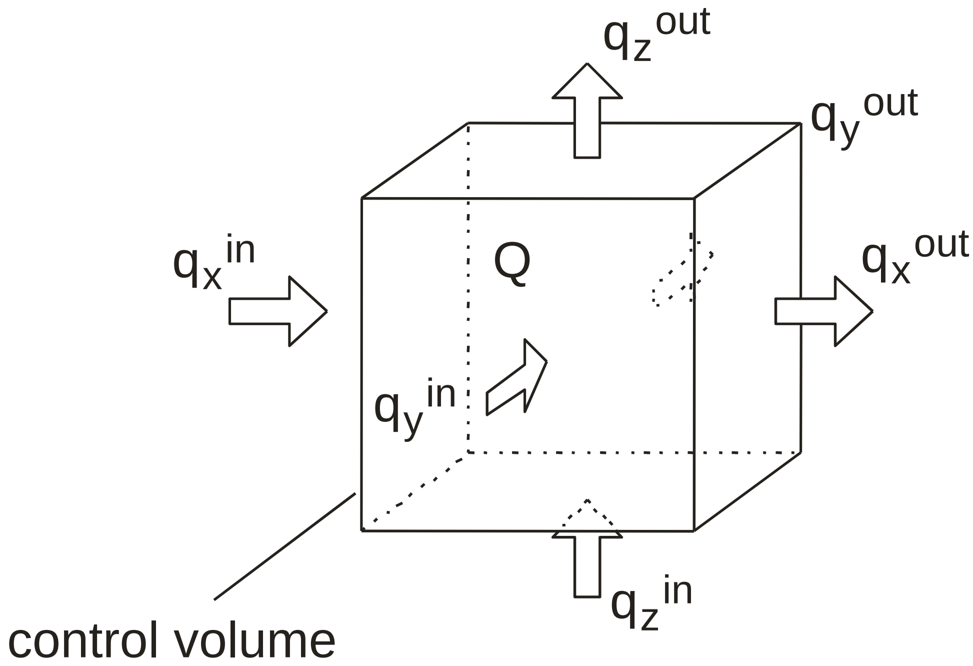
$$q_i = -K_{ij} k_{rel} \frac{\rho \cdot g}{\mu} \frac{\partial}{\partial x_j} \underbrace{\left(\frac{p}{\rho \cdot g} + z \right)}_{h}$$

h	potential head [m]
q_i	Darcy velocity [m/s]
K_{ij}	permeability tensor [m²] $= \begin{bmatrix} k_h & 0 & 0 \\ 0 & k_a & 0 \\ 0 & 0 & k_v \end{bmatrix}$ if anisotropic in direction of the coordinates
$\rho \quad g$	density • gravitation [N/m³]
z	coordinate (direction of gravitation) [m]
k_{rel}	relative permeability [-]
p	pore water pressure [N/m²]

Mass Balance

Steady state flow

Flow into controll volume must be equal to flow out of controll volume (with exception to sinks and sources inside controll volume)



$$\nabla \vec{q} = Q \quad \text{or} \quad \frac{\partial q_i}{\partial x_i} = Q$$

[without sources: Laplace Equation $\nabla^2 h = 0$]

Mass Balance

Transient Flow

For transient processes the water volume of the controll volume can vary.
Water can be stored.

Water flow over the boundary of the controll volume
+ local mass change in time
= sources (+ sinks)

$$\rho \frac{\partial S}{\partial t} + \frac{\partial}{\partial x_i}\left(\rho \cdot q_i\right) = Q \cdot \rho$$

S	storage volume
$\frac{\partial S}{\partial t}$	storage rate
Q	sources if Q>0 sinks if Q<0
q_i	mass flow
ρ	density

Mass Balance

Storage volume contents of:

- compressibility of soil (α)
- compressibility of water (β)
- change of saturation (for unconfined aquifer)

$$\frac{\partial S}{\partial t} = \underbrace{\left(\alpha(1-n) + \beta \cdot n\right)\rho}_{S^{op}} \frac{\partial p}{\partial t}$$

confined aquifer

$$\frac{\partial S}{\partial t} = \left[S^{w}\left(\alpha(1-n) + \beta \cdot n\right) + n \cdot \frac{\partial S^{w}}{\partial p}\right] \cdot \rho \cdot \frac{\partial p}{\partial t}$$

unconfined aquifer

n porosity

S^w saturation

S^{op} specific storage coefficient

in unconfined aquifer is

$$n \cdot \frac{\partial S^{w}}{\partial p} >> S^{w} \cdot S^{op}$$

$\Rightarrow$ storage is dominated by change of saturation

Flow Equation

$$\underbrace{\cancel{\rho}\left(\cancel{S^{op}}\cdot\cancel{S^{w}+n}\cdot\cancel{\frac{\partial S^{w}}{\partial p}}\right)\cdot\cancel{\frac{\partial p}{\partial t}}}_{\dfrac{\partial S}{\partial t}}-\underbrace{\frac{\partial}{\partial x_i}\left[\overbrace{\rho\cdot k_{ij}\cdot\cancel{k_{rel}}\,\frac{\rho\cdot g}{\mu}\cdot\frac{\partial}{\partial x_i}\cdot\left(\underbrace{\frac{p}{\rho\cdot g}+z}_{\mathbf{h}}\right)}^{\mathbf{q_i}\ \textbf{(Darcy)}}\right]}_{\dfrac{\partial}{\partial x_i}\left(\rho\cdot q_i\right)}=\overset{0}{\cancel{Q\rho}}$$

saturated, transient flow

$$\rho\cdot S^{op}\cdot\frac{\partial p}{\partial t}-\frac{\partial}{\partial x_i}\left[\rho\cdot k_{ij}\,\frac{\rho\cdot g}{\mu}\cdot\frac{\partial}{\partial x_j}\cdot\left(\frac{p}{\rho\cdot g}+z\right)\right]=Q\rho$$

saturated steady-state flow

$$-\frac{\partial}{\partial x_i}\left[\rho\cdot k_{ij}\,\frac{\rho\cdot g}{\mu}\cdot\frac{\partial}{\partial x_j}\cdot\left(\frac{p}{\rho\cdot g}+z\right)\right]=Q\rho$$

sourcefree, saturated steady-state flow in homogenous media (k_f = const.)

$$\rho\cdot k_{ij}\,\frac{\rho\cdot g}{\mu}\left[-\frac{\partial}{\partial x_i}\cdot\frac{\partial}{\partial x_j}\cdot\left(\frac{p}{\rho\cdot g}+\right)\right]=0\qquad\Rightarrow\qquad-\frac{\partial}{\partial x_i}\cdot\frac{\partial}{\partial x_j}\cdot h=0\qquad\Rightarrow\qquad\nabla^2 h=0$$

Laplacegleichung

Boundary and Initial Condition

Complete description of flow needs boundary conditions and for transient flow additionaly initial conditions.

boundary conditions

1. kind (Dirichlet): given potential head

$$h = \bar{h}$$

2. kind (Neumann): given boundary flux

$$q_i n_i = \bar{q}$$

3. kind (Cauchy): relationship between flux and potential head (Leakage)

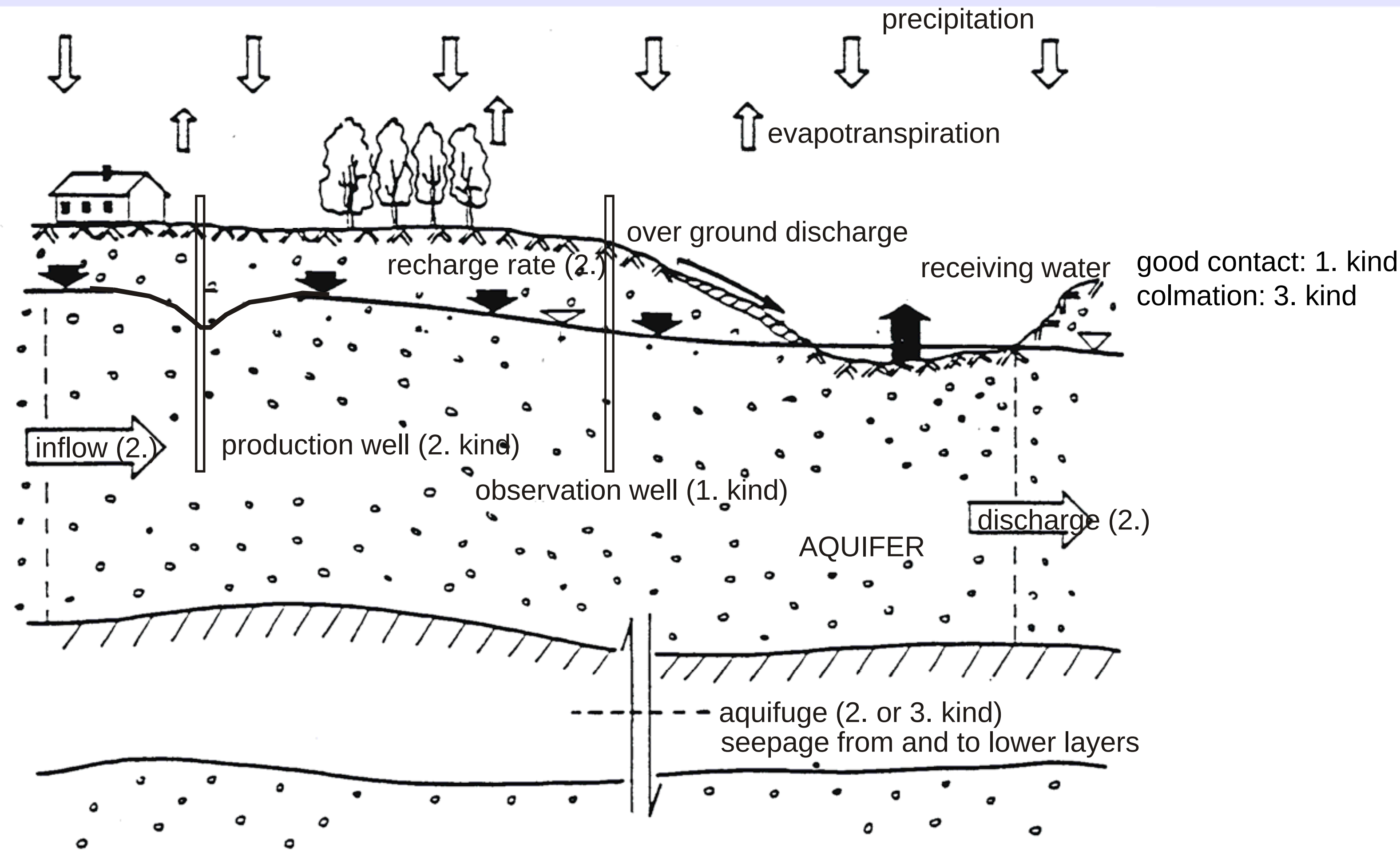
$$q_i n_i = f(\Delta h)$$

initial conditions

potential head for time t=0

$$h(t = 0) = h_0$$

boundary conditions



also: water shed, sheet pile wall, streamline: q=0

standing water body....

recharge rate

NRW: precipitation approx. 800 mm/a, recharge rate averages approx. 200 mm/a

influenced by:

- climate, in particular precipitation
- relief
- land use
- vegetation
- soil type of surface layer
- distance between groundwater table and surface

calculation of recharge rate (e.g. NRW)

JOHANNES MEßER (1996) "Auswirkungen der Urbanisierung auf die Grundwasser-Neubildung im Ruhrgebiet unter besonderer Berücksichtigung der Castroper Hochfläche und des Stadtgebietes Herne" - Dissertation, Math.-Naturwissenschaftliche Fakultät der Technischen Universität Clausthal

SCHROEDER UND WYRWICH (1990) "Eine in Nordrheinwestfalen angewendete Methode zur flächendifferenzierten Ermittlung der Grundwasserneubildung" - Deutsche Gewässerkundliche Mitteilungen, 34, Koblenz 1990

Nonlinearities

Thickness of aquifer depends on groundwater table, which is the unknown variable.

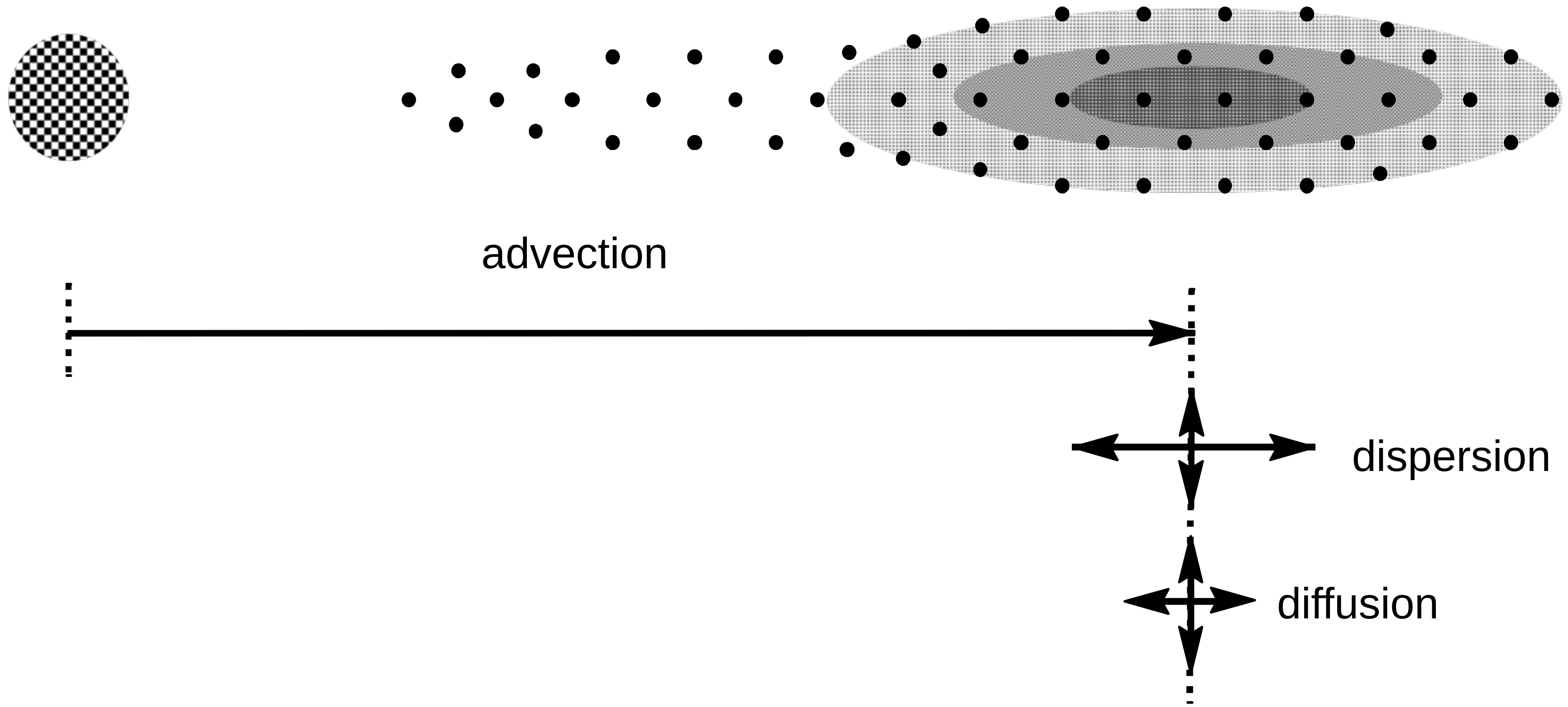
Saturation relations (saturation, pressure, relative permeability) are nonlinear

$\longrightarrow$ Iteration scheme is needed

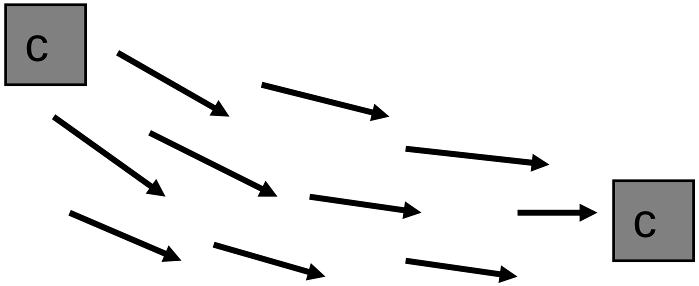
Depending on the grade of nonlinearity a linearization is needed (e.g. Newton Iteration).

In most cases only a simple interation is used (calculation with assumption of starting values, new calculation with improved starting values until accuracy is satisfying or maximum iteration number is reached)

Transport Mechanism in Groundwater



Advection

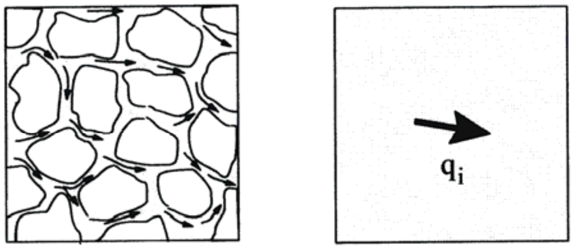


transport with groundwater flow

depends on the average particle velocity
of groundwater (distance velocity)

advectiv flux = particle velocity * concentration

$$j_i^a = u_i c$$



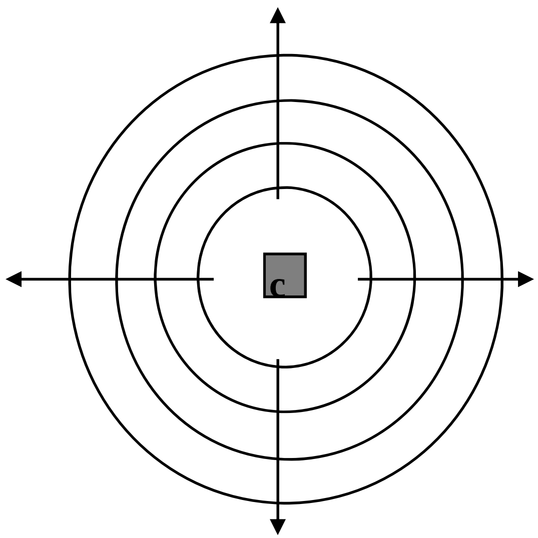
particle velocity darcy velocity

average particle velocity (distance velocity) =
Darcy velocity / effective porosity

$$u_i = \frac{q_i}{n^{eff}}$$

Diffusion

flux from higher to lower concentration



depends on:

- soil characteristics
- gradient of concentration

depends <u>not</u> on:

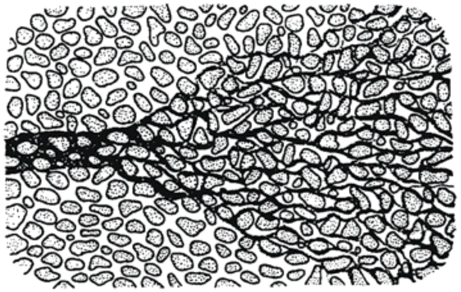
- groundwater flow !
- direction

flux = diffusion coefficient * (- gradient of concentration)

$$j_i^m = -D \frac{\partial c}{\partial x_i}$$

Hydromechanical Dispersion

Spreading of the transported substance due to the way around the grains and heterogenity of permeability in all scales



depends on:
- soil characteristics
- flow velocity
- model scale
- gradient of concentration

flux = dispersion tensor * (- gradient of concentration)

$$j_i^d = -D_{ij}\frac{\partial c}{\partial x_j}$$

$$D_{xx} = \alpha_L \frac{u_x^2}{|u|} + \alpha_{TH}\frac{u_y^2}{|u|} + \alpha_{TV}\frac{u_z^2}{|u|} \qquad D_{xy} = D_{yx} = (\alpha_L - \alpha_{TH})\frac{u_x u_y}{|u|}$$

$$D_{yy} = \alpha_{TH} \frac{u_x^2}{|u|} + \alpha_{L}\frac{u_y^2}{|u|} + \alpha_{TV}\frac{u_z^2}{|u|} \qquad D_{xz} = D_{zx} = (\alpha_L - \alpha_{TV})\frac{u_x u_z}{|u|}$$

$$D_{zz} = \alpha_{TV} \frac{u_x^2}{|u|} + \alpha_{TV}\frac{u_y^2}{|u|} + \alpha_{L}\frac{u_z^2}{|u|} \qquad D_{yz} = D_{zy} = (\alpha_L - \alpha_{TV})\frac{u_y u_z}{|u|}$$

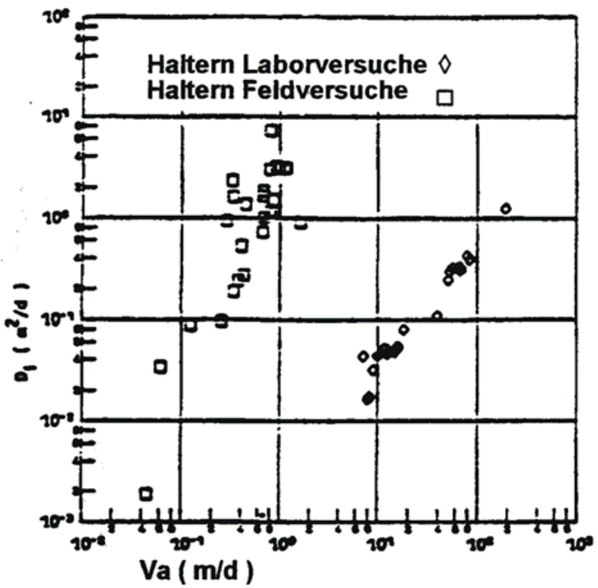
mit: α_L longitudinal dispersion coefficient [m]

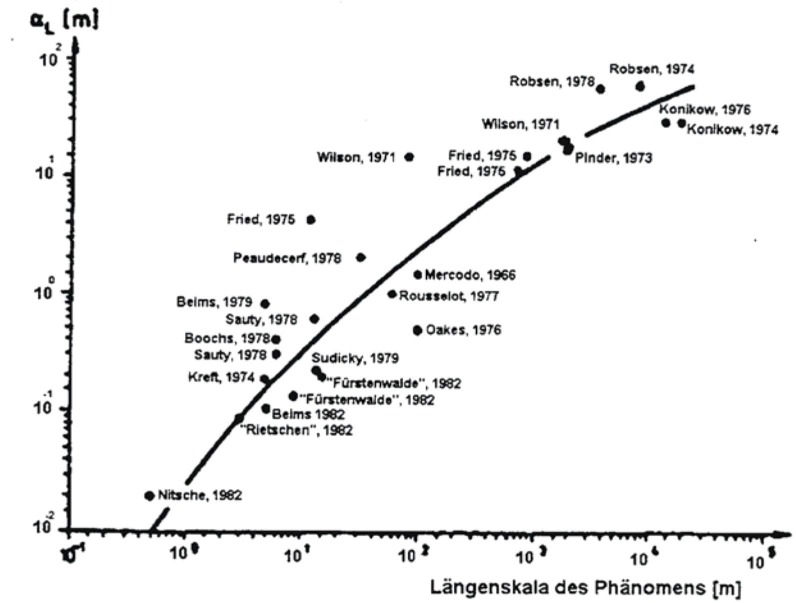
α_{TH} transversal horizontal dispersion coeff. [m]

α_{TV} transversaler vertical dispersion coeff. [m]

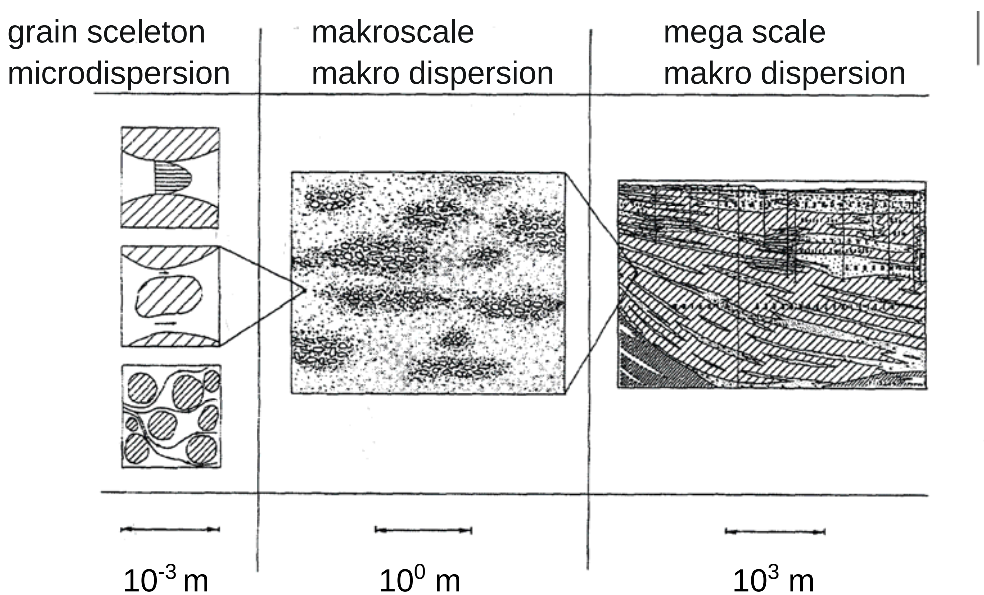
u_i distance velocity [m/s]

Scale Dependency of Dispersion

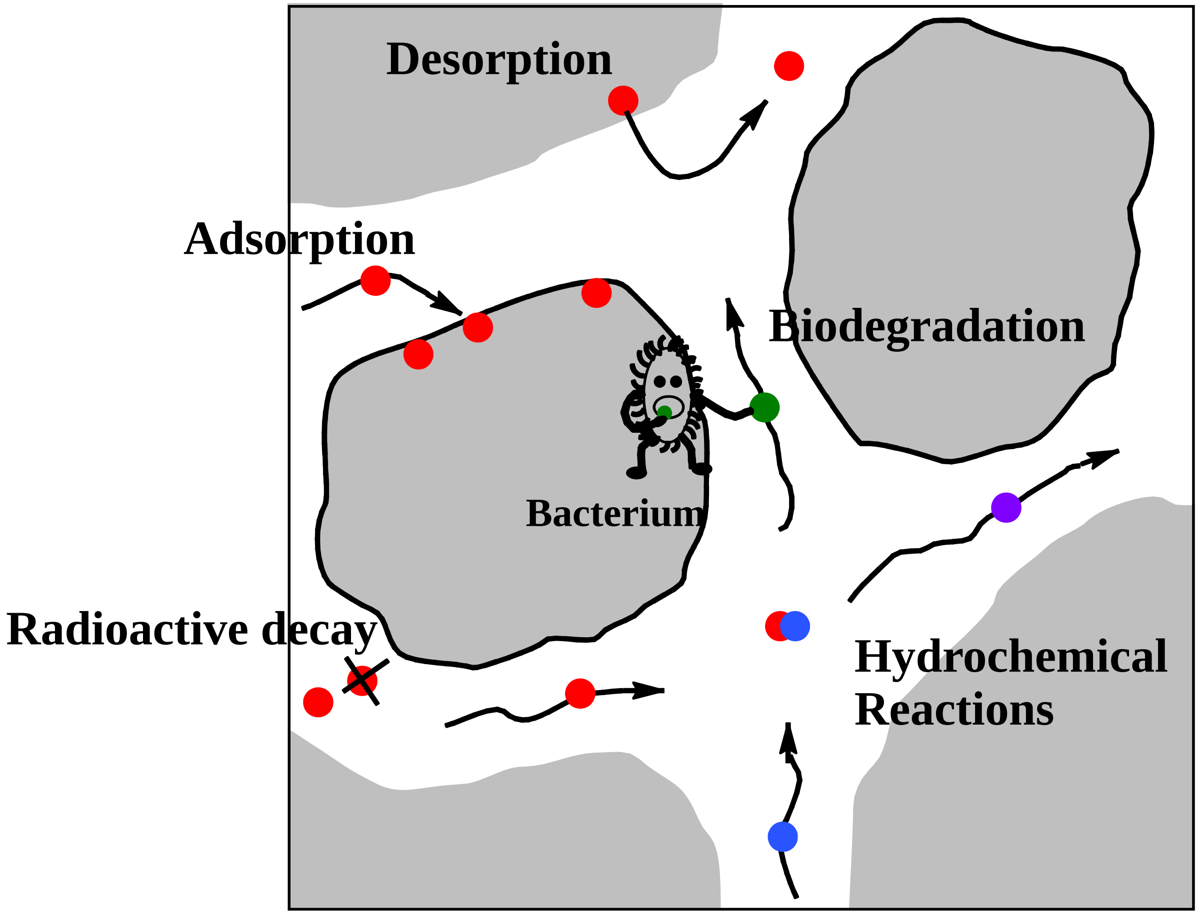




reasons for scale dependency



Nonconservative Transport Mechanism



Transport Equation

(partial differential equation of second order)

- General transport equation (advektion-diffusion-equation)

$$\frac{\partial(cn)}{\partial t} + \frac{\partial}{\partial x_i}(nq_i c) - \frac{\partial}{\partial x_i}\left(n(D_{ij} + d_m \delta_{ij}) \frac{\partial c}{\partial x_j} \right) = 0$$

parabolic hyperbolic type

- pure diffusive transport equation

parabolic type

$$\frac{\partial(cn)}{\partial t} - \frac{\partial}{\partial x_i}\left(nD_{ij} \frac{\partial c}{\partial x_j} \right) = 0$$

continuous change of concentration (as for heat conduction)

- pure advective transport equation

hyperbolic type

$$\frac{\partial(cn)}{\partial t} + \frac{\partial}{\partial x_i}(nq_i c) = 0$$

saltus of concentration at spreading front (as wave propagation)

→ This part causes stability problems in numerical approximation

Transport PDE

boundary and initial conditions

- B.C. 1. kind (Dirichlet)
  given concentration

$$c = \bar{c}$$

- B.C. 2. kind (Neumann)
  dispersive mass flux over the boundary

$$-D_{ij} n_i \frac{\partial c}{\partial x_j} = \overline{j_\perp}$$

For no flow boundaries = 0. Otherwise difficult to specify.

- B.C. 3. kind (Cauchy)
  total flux over the boundary

$$\left(v_a c - D_{ij} \frac{\partial c}{\partial x_j} \right) n_i = \overline{j_\perp}$$

- initial condition
  concentration for time t=0

$$c(t_0) = \overline{c_0}$$